MODELLING OF FLOW OF A LIQUID IN APPARATUS WITH MOBILE PACKING: DISPERSION MODEL

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The paper deals with determination of parameters of a dispersion model used for describing the flow of liquid on a plate with mobile packing in the region of gas velocities up to 1.5 m s^{-1}. The parameters of the model – the diffusion Peclet number and mean residence time of liquid – were determined from the nonideal input impulse of tracer concentration and its response by the method of numerical integration of differential equation with subsequent optimization of the parameters. The results of measurements are presented graphically and in the form of criterion equations.

The main part of an apparatus with mobile packing is a three-phase fluidized layer formed on a plate with a bed of spherical particles as a consequence of high velocities of both gas and liquid. Although apparatuses of this type have found applications in a number of chemical and other technologies, the processes taking place in the bed have not been described adequately yet. The operation of an apparatus with mobile packing is characterized by high velocities of gas (2 – 5 m s^{-1}) and liquid. A suitable choice of characteristics of the plate and packing, however, enables – even in the region of lower gas velocities – realization of regimes in which there are good conditions for heat and mass transfer.

The basic information necessary for evaluation of experiments concerning heat and/or mass transfer involve quantitative data about the degree of mixing of the liquid. The modelling of flow of liquid on a plate with mobile packing was dealt with in a number of communications[1 – 11] which, however, form an only small part of the total number of papers published in the area of hydrodynamics and heat and mass transfer.

For an apparatus with mobile packing the model most frequently used for the flow of liquid is the dispersion model[1 – 4, 6 – 9, 11] whose parameters can be determined by a transient response technique. This model is represented by a partial differential equation describing the dependence of tracer concentration on time and place.

The aim of the present work is to experimentally determine the intensity of mixing of liquid on a plate with mobile packing in the gas velocity region below 1.5 m s^{-1} which has not been systematically investigated yet.

EXPERIMENTAL

The laboratory apparatus used consisted of a column of 0.109 m inner diameter with one plate without downcomer with 52% free area. The mobile packing was formed by polystyrene spheres of $9 \cdot 10^{-3}$ m diameter and 1 000 kg m^{-3} density. The spheres were introduced on the plate in layers. For the measurements we used 4, 6, and 9 layers corresponding to the static bed heights of $33 \cdot 10^{-3}$, $49 \cdot 10^{-3}$, and $74 \cdot 10^{-3}$ m, respectively. Tap water and air (from pressure air supply) were used as the liquid and gas phases, respectively. The velocity of liquid related to the column cross section was equal to $5.36 \cdot 10^{-3}$, $8.93 \cdot 10^{-3}$, and $12.50 \cdot 10^{-3}$ m s^{-1}, the gas velocity was varied in the limits from 0.29 to 1.41 m s^{-1}. A schematic representation of the apparatus used is given in ref.[11].

The nature of experimental work required a determination of the height of expanded bed. This quantity was estimated visually as the mean value of the highest occurring particles during a 10 min interval.

The tracer used in the experiments focused on quantitative determination of the degree of mixing of the liquid was an almost saturated KCl solution which was injected into the water stream before the distributor of liquid. Its concentration in both the inlet and outlet streams was determined conductometrically. One conductometric cell was placed at the end of the distributor whose height above the plate was adjusted at a value corresponding to the height of the expanded bed. The second cell was placed immediately below the plate. The signals from the conductometers (type OK-102/1, firm Radelkis), proportional to the KCl concentrations in each stream, were introduced into a twelve-bit A/D converter interfaced with a PC/AT 286 computer. The levels of digital signals were recalculated to concentration data (on the basis of calibration of each conductometric cell) and saved in data files. The time dependence of tracer concentration at the inlet of the bed was realized as a nonideal input impulse. After starting the program for data recording we opened the supply of KCl dosed from a pressure storage container through a capillary and monitored the KCl concentrations in both the streams. After reaching the same concentration in the two streams the KCl supply was stopped. The data sampling was finished when the KCl concentration attained zero values in both streams. The KCl flow rate was chosen in such a way as to ensure a sufficient conductivity change and not to affect significantly the overall flow of liquid. In all the measurements the sampling frequence was constant – 5 Hz, and ca 200 – 1 000 data triads (time, KCl concentrations at the inlet and at the outlet of the bed) were obtained during each measurement. Each measurement was repeated five times at least, and the data sets obtained were treated independently.

TREATMENT OF RESULTS OF MEASUREMENTS

The parameters of dispersion model, i.e. the diffusion Peclet number *Pe* and the mean residence time of liquid, τ_m, were determined separately from the data sampling, by the procedure described in ref.[11] which can be summarized in the following steps:

1. The starting estimate of the Peclet number and mean residence time of liquid (in all the experiments $Pe^{(0)} = 5$, $\tau_m^{(0)} = 7$ s).

2. The numerical integration of the partial differential equation describing the dispersion in the system. The integration was carried out by the Crank–Nicolson method with the integration steps of $0.2/\tau_m$ in the direction of time and the dimensionless longitudinal coordinates 0.05. In this step one obtains the calculated values of dimensionless concentration of tracer (the ratio of actual and the maximum concentrations) in the outlet stream of liquid.

3. The calculation of the objective function defined as a sum of squares of deviations between the experimental and the calculated values of dimensionless concentration of tracer in the outlet stream.

4. The realization of one step of minimization of the objective function by the modified simplex method – the alghorithm by Nelder and Mead[12]. In this step one obtains the corrected values of *Pe* and τ_m.

5. The procedure starting from step 2 was repeated until reaching the minimum of the objective function defined in step 3.

The suitability of the procedure used can be evaluated from Fig. 1 giving the time dependences of the dimensionless KCl concentrations at the inlet and outlet of the bed along with the theoretical curve according to the dispersion model. The agreement between both output signals indicates a good agreement between experiment and model.

RESULTS AND DISCUSSION

Bed Expansion

From the experimentally found height values of expanded bed we calculated the degree of bed expansion h_d/h_0 for the given conditions. These values are treated graphically in Figs 2 and 3 giving the plot of dependence of expansion degree on the Reynolds number for gas for both limit values of the simplex h_0/D_c = 0.30 and 0.68. The Reynolds number for liquid is the parameter of the lines. From both figures it is obvious that the degree of bed expansion increases with increasing value of the Reynolds number for both the gas and the liquid. The expansion degree diminishes with increasing value of the simplex h_0/D_c. The same conclusions about the effects of Re_G and Re_L on the bed expansion degree are arrived at also when evaluating the corresponding dependences

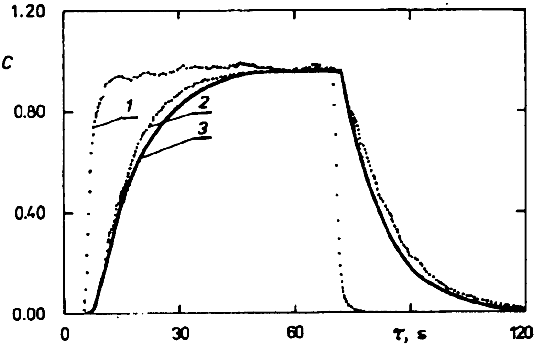

FIG. 1
Time dependence of concentration of tracer: *1* experimental values (inlet); *2* experimental values (outlet); *3* calculated values (outlet)

for $h_0/D_c = 0.45$. The lines in Figs 2 and 3 represent the expansion degrees calculated from Eq. (*1*) which was used for correlations of all experimental data concerning the expansion of layer

$$h_d/h_0 = 2.56 \cdot 10^{-2} Re_G^{0.126} Re_L^{0.562} \left(\frac{h_0}{D_c}\right)^{-0.417} . \quad (1)$$

The applicability of this equation, whose constant and exponents were determined by nonlinear regression, can be judged from the value of the mean quadratic relative error calculated from Eq. (2)

$$\bar{\delta} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{(h_d/h_0)_{i,\exp} - (h_d/h_0)_{i,\text{calc}}}{(h_d/h_0)_{i,\exp}}\right)^2} \cdot 100 . \quad (2)$$

This error is 4.7% ($n = 75$).

Diffusion Peclet Number

Figure 4 presents the dependence of the diffusion Peclet number upon the Reynolds number for gas for $Re_L = 744$. The simplex h_0/D_c is the parameter of the lines. From the figure it is obvious that in the Reynolds number interval investigated ($Re_G = 2.2 \cdot 10^3 - 10.2 \cdot 10^3$) the Peclet number is slightly increased with increasing Re_G value. The increase in the simplex h_0/D_c value from 0.30 to 0.45 results in a minor decrease in the Peclet number, whereas further increasing h_0/D_c leads to larger changes in the *Pe* values. The same conclusions can be arrived at also from the evaluation of the effect of Re_G and h_0/D_c on the Peclet number even for $Re_L = 447$. At the highest value, $Re_L =$

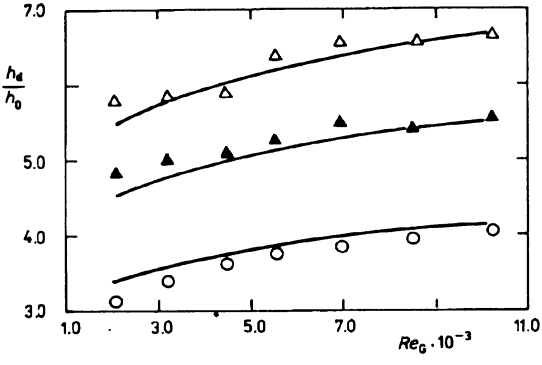

FIG. 2
Dependence of degree of bed expansion on Reynolds number for gas at $h_0/D_c = 0.30$: ○ $Re_L = 447$; ▲ $Re_L = 744$; Δ $Re_L = 1\ 042$

1 042, the conclusions about the effect of Re_G on the Peclet number are identical with those valid for Re_L = 447 and 744. The change in the value of simplex h_0/D_c from 0.3 to 0.45 results in a significant decrease of Pe, whereas a further increase in the value of simplex h_0/D_c causes an only slight decrease of the Peclet number.

The effect of Re_L upon Pe can be evaluated from Fig. 5 giving the dependence $Pe = f(Re_G)$ for the constant value of simplex h_0/D_c = 0.45. In this case the parameter of lines is the Reynolds number for liquid (Re_L = 447, 744, and 1 042; for Re_L = 447 and 744 the dependence $Pe = f(Re_G)$ is approximated by a single line). It can be seen that the increase in Re_L from 447 to 744 does not significantly affect the values of the Peclet number; the Pe value only diminishes when going from Re_L = 744 to Re_L = 1 042. The same conclusions about the influence of Re_L on the Peclet number can also be made for the value of h_0/D_c = 0.3. In the case of the highest value used for static bed height, h_0 = 74 . 10^{-3} m (h_0/D_c = 0.68), it was found that the Peclet number was independent of Re_L.

The equation (*3*), similar in form to Eq. (*1*), was suggested for correlating the experimental values of diffusion Peclet number

$$Pe = 0.522\, Re_G^{0.232}\, Re_L^{-0.267} \left(\frac{h_0}{D_c}\right)^{-0.702} . \qquad (3)$$

Again the constant and exponents of Eq. (*3*) were determined by nonlinear regression. The adequacy of correlation (*3*) was judged on the basis of the mean quadratic relative error which is 36.1% for a set of 431 experimental data. The high value of mean quadratic relative error is due to relatively large deviations in the Pe values determined. A similar scattering in correlating experimental data about mixing on a plate with mobile packing was also reported in ref.[6].

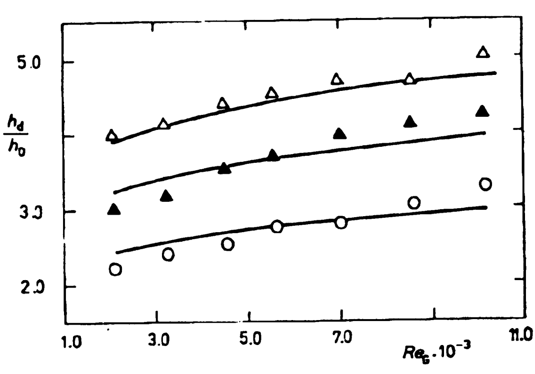

FIG. 3
Dependence of degree of bed expansion on Reynolds number for gas at h_0/D_c = 0.68: ○ Re_L = 447; ▲ Re_L = 744; Δ Re_L = 1 042

The conslusions about the effect of the flow rate of liquid upon the Peclet number are qualitatively identical with those by other authors[4,6,8], and we can observe an excellent agreement between the exponent of Re_L and the value of exponent of u_L in ref.[8] viz. −0.28 (though the correlation given[8] concerns the flow rates of gas above 1.5 m s^{-1}). Uysal and Ozilgen[9] and Chen and Douglas[2] (in the latter paper the results are evaluated with the ratio of (Pe'/Pe_0')) did not found any effect of the velocity of liquid upon the Peclet number. Moreover, in ref.[9] it was found that from among the other quantities followed (u_G, h_0, and d_P) the Peclet number only depends on the diameter of particles of packing. The Peclet number value determined from the correlation[9] and recalculated to the height of contact space is equal to 1.0 for particles of 15 . 10^{-3} m diameter, which value is comparable with the *Pe* values found in the present paper although the working conditions characterized by the velocity of gas and other parameters are different. As far as the effect of the velocity of gas on the diffusion Peclet number is concerned, the results obtained stand in contrast with conclusions of a number of papers[2,4,6,8], where the decrease in the Peclet number with increasing velocity of gas was unambiguously proved. In the case of evaluation of the effect of static bed height upon the Peclet number there exists a qualitative agreement with the paper by Wasowski and Mlodzinski[8] who (in contrast to ref.[6]) found a decrease in the Peclet number with increasing static bed height. The mean *Pe* values in ref.[8] are 1.5 ÷ 2.5 for the region of developed fluidization and correspond to those in the present paper.

Mean Residence Time of Liquid

A part of the results of measurements concerning the mean residence time of liquid on the plate, τ_m, whose values were obtained along with the values of Peclet number are presented in Figs 6 and 7 giving the plot of the dependence of mean residence time of

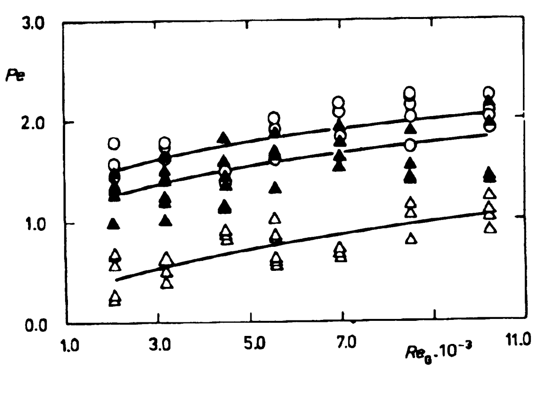

FIG. 4
Dependence of diffusion Peclet number on Reynolds number for gas at Re_L = 744: ○ h_0/D_c = 0.30; ▲ h_0/D_c = 0.45; Δ h_0/D_c = 0.68

liquid upon the Reynolds number for gas. The parameters of lines in Figs 6 and 7 are the simplex h_0/D_c and Re_L, respectively. From the figures given it follows that the mean residence time of liquid on the plate decreases with increasing gas velocity, this decrease being particularly marked in the region of low Re_G values ($Re_G < 4\ 500$). Furthermore it is obvious that τ_m increases with increasing value of simplex h_0/D_c and decreases with increasing Re_L value. The same conclusions can be arrived at also in evaluating the dependences $\tau_m = f(Re_G)$ for other combinations of Re_L and h_0/D_c.

The curves in Figs 6 and 7 were obtained by the regression of corresponding data sets for Re_L = const. and h_0/D_c = const. using the second-order polynomial:

$$\tau_m = a_0 + a_1 Re_G + a_2 Re_G^2 , \tag{4}$$

whose constants a_0, a_1, and a_2 are given in Table I.

The equation (5) was suggested for correlating all the experimental data concerning the mean residence time,

$$U_L = A\ Re_G^{p_1}\ Re_L^{p_2} \left(\frac{h_0}{D_c}\right)^{p_3} , \tag{5}$$

where U_L is the dimensionless velocity of liquid in the bed which is defined by Eq. (6) and can be interpreted physically as the reciprocal value of dimensionless hold-up of liquid

$$U_L = \bar{u}_L/u_L = h_d/(\tau_m u_L) . \tag{6}$$

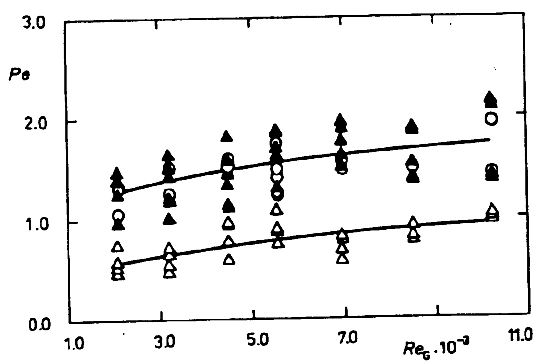

FIG. 5
Dependence of diffusion Peclet number on Reynolds number for gas at h_0/D_c = 0.45: ○ Re_L = 447; ▲ Re_L = 744; Δ Re_L = 1 042

Using the fitted values of expanded bed height from correlation (*1*), we determined the constant and exponents of Eq. (*5*) by regression. The correlation has the following form:

$$U_L = 5.68 \cdot 10^{-2} Re_G^{0.470} Re_L^{-0.032} \left(\frac{h_0}{D_c}\right)^{-0.068} . \qquad (7)$$

The value of mean quadratic relative error is $\bar{\delta} = 7.2\%$ for the set of 431 measurements. With regard to the low sensitivity of dimensionless velocity of liquid in bed to the Reynolds number for liquid (Fig. 8) and to the simplex h_0/D_c, we moreover determined the constant and exponent of the simplified Eq. (*5*); the final form of the correlation reads as follows:

$$U_L = 4.84 \cdot 10^{-2} Re_G^{0.472} . \qquad (8)$$

TABLE I
Constants of polynomial (*4*) (in s)

Re_L	$h_0/D_c = 0.30$			$h_0/D_c = 0.45$			$h_0/D_c = 0.68$		
	a_0	$-a_1 \cdot 10^3$	$a_2 \cdot 10^8$	a_0	$-a_1 \cdot 10^3$	$a_2 \cdot 10^8$	a_0	$-a_1 \cdot 10^3$	$a_2 \cdot 10^8$
447	12.37	0.92	3.86	16.01	1.54	8.06	21.04	2.11	12.09
744	12.68	1.50	7.46	13.84	1.32	6.29	20.33	2.35	12.63
1 042	11.77	1.34	5.54	13.00	1.14	4.23	17.16	1.85	10.08

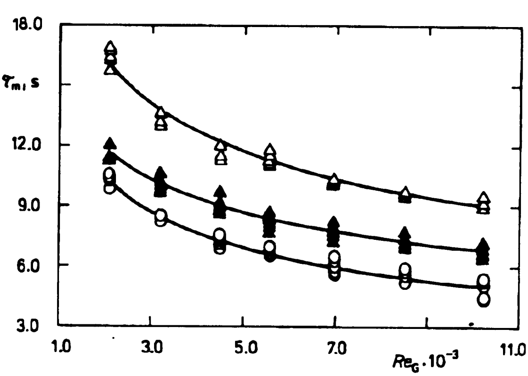

FIG. 6
Dependence of mean residence time of liquid on Reynolds number for gas at $Re_L = 744$: ○ $h_0/D_c = 0.30$; ▲ $h_0/D_c = 0.45$; Δ $h_0/D_c = 0.68$

The mean quadratic relative error is $\bar{\delta}$ = 7.7%.

The values of dimensionless velocity of liquid calculated from Eq. (*8*) are represented by solid line in Fig. 8.

Dispersion Coefficient

The dispersion coefficients D_L were calculated from mean values of the Peclet number, mean residence time of liquid on plate, and experimental values of expanded bed height. A typical dependence of dispersion coefficient on the velocity of gas is presented in Fig. 9. The parameter of lines is the velocity of liquid. The dependence is valid for the constant static bed height $h_0 = 74 \cdot 10^{-3}$ m. The lines represent the values of dispersion coefficient calculated from Eq. (*9*) obtained from the definition equation for *Pe* by combination with the correlation expressions (*1*), (*3*), (*8*), and Eq. (*6*):

$$D_L = 2.37 \cdot 10^{-3} Re_G^{0.366} Re_L^{0.829} \left(\frac{h_0}{D_c}\right)^{0.285} u_L h_0 \,. \qquad (9)$$

From Fig. 9 it can be seen that the experimental values agree well with the values calculated from Eq. (*9*) for the given arrangement. For some combinations of velocity of liquid and static bed height we found greater differences between the experimental and calculated D_L values – the mean quadratic relative error in comparing the given data sets is $\bar{\delta}$ = 17.2%. From Eq. (*9*) and Fig. 9 it is obvious that the intensity of mixing of liquid on the plate with mobile packing characterized by the dispersion coefficient is increased with increasing velocities of gas and liquid as well as increasing static bed height, the values of dispersion coefficient varying within the limits from ca $0.8 \cdot 10^{-3}$ to $20 \cdot 10^{-3}$ m^2 s^{-1}.

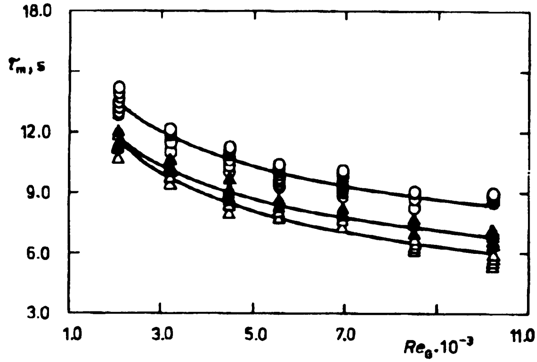

FIG. 7

Dependence of mean residence time of liquid on Reynolds number for gas at h_0/D_c = 0.45: O Re_L = 447; ▲ Re_L = 744; Δ Re_L = 1 042

The qualitative evaluation of the effect of velocities of gas and liquid and of static bed height on the dispersion coefficient is in accordance with the results obtained by Koval et al.[4] and Wasowski and Mlodzinski[8]. From the graphical dependences of $D_L = f(u_G)$ presented in ref.[4] for $h_0 = 80 \cdot 10^{-3}$ m, $d_P = 16 \cdot 10^{-3}$ m, $\rho_P = 283$ kg m^{-3}, and $D_c = 0.1$ m we can estimate the D_L value of about $2.3 \cdot 10^{-3}$ m^2 s^{-1} ($u_G = 1.5$ m s^{-1}, $u_L = 15 \cdot 10^{-3}$ m s^{-1}), which value is approximately ten times smaller than the D_L value found in the present paper for the maximum values of velocities of gas and liquid and static bed heights. It can be presumed that this difference is due to considerably different densities of the packing particles. The values of Peclet number for the conditions given in ref.[4] are considerably affected by the velocity of liquid (up to the gas velocities of 1.5 m s^{-1}): the *Pe* values read from the graph for $u_L = 3.9 \cdot 10^{-3}$, $9.3 \cdot 10^{-3}$, and $15.5 \cdot 10^{-3}$ m s^{-1} are 5.2, 4.0, and 2.5, respectively. Extremely high values of dispersion coefficient, $5.6 \cdot 10^{-3} - 250 \cdot 10^{-3}$ m^2 s^{-1}, were obtained in ref.[1]; the corresponding interval of the Peclet numbers is 0.11 – 0.43. These data indicate very intensive mixing of liquid on plate. Rama et al.[6] found that the dispersion coefficient increases with increasing velocities of both gas and liquid, but it decreases with increasing static bed height. The value of dispersion coefficient (for irregular particles with the sphericity of 0.675) is $6 \cdot 10^{-3}$ m^2 s^{-1} for comparable conditions defined by the gas velocity $u_G = 1.5$ m s^{-1}, liquid velocity $u_L = 11 \cdot 10^{-3}$ m s^{-1}, and the value of simplex $h_0/D_c = 0.78$. This value is considerably lower than the D_L value in the present paper. The large difference between the D_L values could be caused by the difference in the densities of the particles used in the packing: the density of particles in ref.[6] was 112 kg m^{-3}. In ref.[2] – in accordance with ref.[6] – it was found that the dispersion coefficient increases with increasing velocities of gas and liquid. The effect of static bed height on the intensity of mixing of liquid on plate with packing was not investigated.

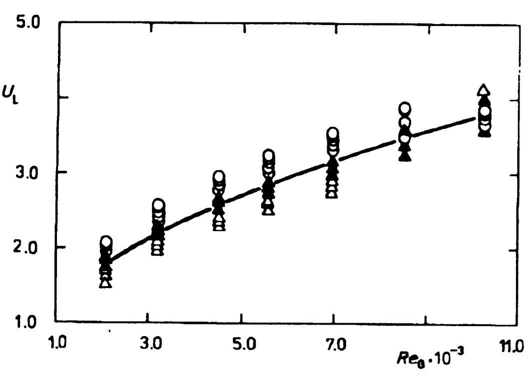

FIG. 8

Dependence of dimensionless velocity of liquid in layer on Reynolds number for gas at $h_0/D_c = 0.68$: ○ $Re_L = 447$; ▲ $Re_L = 744$; Δ $Re_L = 1\ 042$

The comparison of dispersion coefficient as well as that of diffusion Peclet number with the published data must be considered only preliminary because the hydrodynamic conditions in the region of gas velocities up to 1.5 m s^{-1} with application of packing particles with densities close to that of water can differ from the conditions under which these data were obtained in the communications cited.

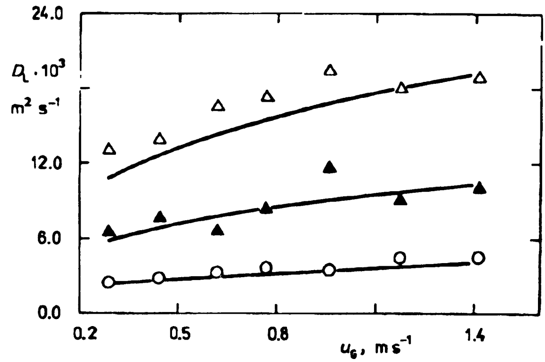

FIG. 9
Dependence of dispersion coefficient on velocity of gas for h_0 = 74 . 10^{-3} m: ○ u_L = 5.36 . 10^3 m s^{-1}; ▲ u_L = 8.93 . 10^{-3} m s^{-1}; Δ u_L = 12.5 . 10^{-3} m s^{-1}

SYMBOLS

A	constant in Eq. (5)
a_i	(i = 0, 1, 2) constants in Eq. (4), s
C	dimensionless concentration, $C = c/c_{max}$
c	molar concentration, kmol m^{-3}
D_c	diameter of column, m
D_L	dispersion coefficient, m^2 s^{-1}
D_{L0}	dispersion coefficient at zero velocity of gas, m^2 s^{-1}
d	diameter, m
h_0	static bed height, m
h_d	expanded bed height, m
n	number of measurements
p_i	(i = 1, 2, 3) exponents
Pe	diffusion Peclet number, $Pe = \bar{u}_L h_d / D_L$
Pe'	diffusion Peclet number, $Pe' = \bar{u}_L d_P / D_L$
Pe_0'	diffusion Peclet number for zero velocity of gas, $Pe_0' = \bar{u}_L d_P / D_{L0}$
Re	Reynolds number, $Re = u D_c / \nu$
U_L	dimensionless velocity defined in Eq. (6)
u	velocity related to cross section of column, m s^{-1}
$\bar{u}_L$	mean velocity of liquid in bed calculated from mean residence time of liquid and expanded bed height, m s^{-1}

$\bar{\delta}$	mean relative quadratic error, %
ν	kinematic viscosity, $m^2 s^{-1}$
ρ	density, kg m^{-3}
τ	time, s
τ_m	mean residence time of liquid, s

Indexes

calc	calculated value
exp	experimental value
G	related to gas
L	related to liquid
max	maximum value
P	related to packing

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